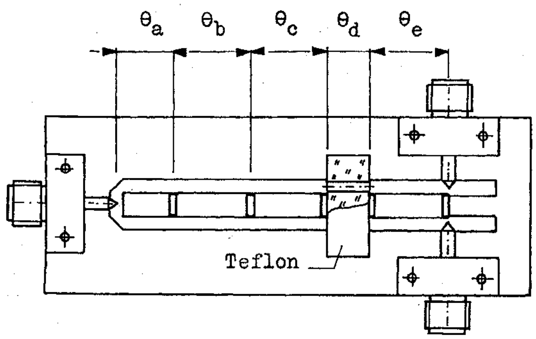
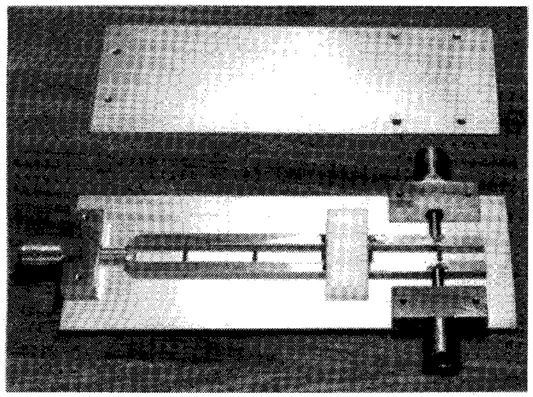


(a)



(b)

Fig. 4. Four-section power divider tested experimentally. (a) Air triplate line configuration. (b) Outside view.

band divider composed of four sections should be designed. The characteristic impedances of those line sections are limited by extreme impedances $Z_{0\min}$ and $Z_{0\max}$, which are assumed freely at the beginning of the design process [8]. Undoubtedly, that opportunity makes their realization easier. Moreover, in the analysis of the even- and odd-mode two-ports, only one type of step discontinuity has to be taken into account. It is justified by the fact that they are equivalent to each other. The compensation of these discontinuities can be achieved by means of the conventional techniques, for instance, as described in [12], [14], and [15]. Also, the parasitic shunt reactances of the isolating resistors can be easily included in the analysis of the odd-mode two-ports. In the first approximations, they are connected in parallel with the half-resistances being sought [see Figs. 1(c) and 2(c)]. The effective two-stage optimization procedure for calculating the isolating resistors is proposed. Presented numerical and experimental results confirm the validity of the proposed design algorithm and indicate that microwave dividers of this type may be adequate for some practical applications.

REFERENCES

[1] E. J. Wilkinson, "An n-way hybrid power divider," *IRE Trans. Microwave Theory Tech.*, vol. MTT-8, pp. 116–118, Jan. 1960.

[2] L. Parad and R. L. Moynihan, "Split-tee power dividers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, pp. 91–95, Jan. 1965.

[3] S. B. Cohn, "A class of broadband three-port TEM-mode hybrids," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 110–116, Feb. 1968.

[4] N. Nagai, E. Maekawa, and K. Ono, "New n-way hybrid power dividers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 1008–1012, Dec. 1977.

[5] R. B. Ekinge, "A new method of synthesizing matched broadband TEM-mode three ports," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 81–88, Jan. 1971.

[6] S. Rosloniec and J. Kleinknecht, "Impedance transforming hybrid power dividers," *Archiv fur Elektronik und Ubertragungstechnik*, vol. AEU-47, pp. 270–272, July 1993.

[7] J. Reed and G. Wheeler, "A method of analysis of symmetrical four-port networks," *IRE Trans. Microwave Theory Tech.*, vol. MTT-4, pp. 246–252, Oct. 1956.

[8] S. Rosloniec, "Design of stepped transmission line matching circuits by optimization methods," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-42, pp. 2255–2260, Dec. 1994.

[9] V. F. Demyanov, "Algorithms for some minimax problems," *J. Comput. Syst. Sci.*, vol. 2, pp. 342–380, Dec. 1968.

[10] D. M. Himmelblau, *Applied Nonlinear Programming.* New York: McGraw-Hill, 1972.

[11] M. S. Bazaraa and C. M. Shetty, *Nonlinear Programming, Theory and Algorithms.* New York: Wiley, 1979.

[12] S. Rosloniec, *Algorithms for Computer-Aided Design of Linear Microwave Circuits.* Norwood, MA: Artech, 1990.

[13] B. C. Wadell, *Transmission Line Design Handbook.* Norwood, MA: Artech, 1991.

[14] L. G. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance Matching Networks and Coupling Structures.* New York: McGraw-Hill, 1964.

[15] K. C. Gupta and R. Chadha, "Design real-word stripline circuits," *Microwaves*, vol. 17, pp. 70–80, Dec. 1978.

Field Distributions in Supported Coplanar Lines Using Conformal Mapping Techniques

N. H. Zhu, E. Y. B. Pun, and P. S. Chung

Abstract—**We use conformal mapping techniques to derive the analytical expressions for calculating the field distributions in supported coplanar lines. Our calculations agree well with the results obtained using the point matching method. Our method is an extension of the approximate technique proposed by Veyres and Hanna [1], and provides an accurate and fast calculation of the field distributions. This method can be extended to the analysis of other coplanar lines widely used in monolithic microwave integrated circuits (MMIC) and optical integrated circuits (OIC) applications.**

I. INTRODUCTION

Coplanar lines have been investigated extensively in monolithic microwave integrated circuits (MMIC's). The quasistatic analyzes of various coplanar lines are well studied using the conformal mapping techniques [1]–[8]. The efforts have been directed mainly toward deriving the approximate analytical formulas for quasistatic parameters, such as effective dielectric constant and characteristic impedance. The point matching method [9] can be used to calculate the field distributions. Unfortunately, its solutions display serious

Manuscript received April 9, 1995; revised April 19, 1996. This work is supported in part by Science Foundation of Guangdong Province, P. R. China, the Croucher Foundation, and a Strategic Research Grant, City University of Hong Kong.

N. H. Zhu is with the Department of Electronic Engineering, City University of Hong Kong, Kowloon, Hong Kong, on leave from Zhongshan University, Guangdong, P. R. China.

E. Y. B. Pun and P. S. Chung are with the Department of Electronic Engineering, City University of Hong Kong, Kowloon, Hong Kong.

Publisher Item Identifier S 0018-9480(96)05648-7.

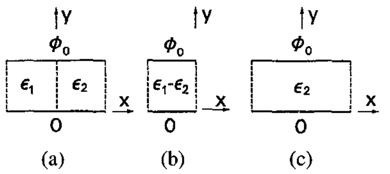


Fig. 1. (a) Parallel plate capacitor, (b) and (c) two resolved parallel plate capacitor with magnetic or electric walls only.

oscillations at the electrode plane. The field distributions can be calculated using the well-known numerical methods, such as the finite element method, the method of lines, and the boundary element method [10]. However, the analytical solutions are more attractive for the engineers engaged in the CAD of MIC's and MMIC's. In this paper, we derive, for the first time, the analytical expressions for the field distributions in two typical examples of supported coplanar lines. The analytical results agree well with those obtained using the point matching method [9].

II. Formulation

Let us first consider the parallel plate capacitor shown in Fig. 1(a). Because the dielectric interface is a magnetic wall, this structure can be divided into two parallel plate capacitors, as shown in Fig. 1(b) and 1(c). The electric fields of the three structures are identical, and can be expressed as $\vec{E} = -a_y\phi_0/d$ where $\vec{a}_y$ is the unit vector in the y direction, ϕ_0 is the applied voltage, and d is the distance between the two parallel electrodes. From the solutions of the two resolved structures, the electric displacement in Fig. 1(a) can also be written as

$$\vec{D} = \begin{cases} (\epsilon_1 - \epsilon_2)\vec{E} + \epsilon_2\vec{E} = \epsilon_1\vec{E}, & x < 0 \\ \epsilon_2\vec{E}, & x > 0. \end{cases} \tag{1}$$

Hence, the same electric field can be obtained since the structure shown in Fig. 1(a) can be divided into two parallel structures.

Next, we consider the coplanar waveguide (CPW) on two-layered dielectric substrate as shown in Fig. 2(a). If the supporting material is replaced by air, this structure is reduced to the coplanar waveguide with finite substrate thickness. Based on the assumptions made in [1], the structure can be investigated by resolving it into two configurations as shown in Fig. 2(b) and 2(c), with magnetic or electric walls only. By successive conformal mapping procedures, the field distribution produced by the structure in Fig. 2(b) can be written as

$$\vec{E}_a = -\frac{\phi_0}{K(k_1')}[\vec{a}_x \mathrm{Im}(dW_1/dZ) + \vec{a}_y \mathrm{Re}(dW_1/dZ)] \tag{2a}$$

where

$$k_1 = \sinh(\pi a/2h)/\sinh(\pi b/2h), \tag{2b}$$

$$k_1' = \sqrt{1 - k_1^2}, \tag{2c}$$

$$dW_1/dZ = \frac{(\pi/2h)\cosh(\pi Z/2h)}{\sinh(\pi a/2h)\sqrt{1 - Z_1^2)(1 - k_1^2 Z_1^2)}}, \tag{2d}$$

$$Z_1 = \sinh(\pi Z/2h)/\sinh(\pi a/2h) \tag{2e}$$

$K(k)$ is the complete elliptic integral of the first kind, $\vec{a}_x$ and $\vec{a}_y$ are the unit vectors in the x and y directions, respectively. The field produced by the structure shown in Fig. 2(c) can be expressed as [11]

$$\vec{E}_b = \frac{\phi_0}{K(k')}[\vec{a}_x \mathrm{Im}(dW/dZ) + \vec{a}_y \mathrm{Re}(dW/dZ)] \tag{3a}$$

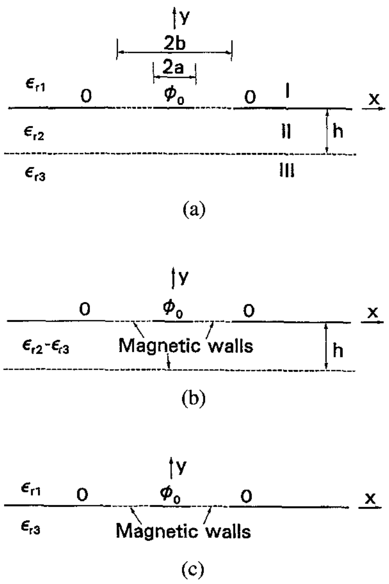


Fig. 2. (a) Supported coplanar waveguide, (b) and (c) two resolved structures for calculating the field distributions.

where

$$k = a/b, \tag{3b}$$

$$dW/dZ = \frac{(1/a)}{\sqrt{[1 - (Z/a)^2][1 - (Z/b)^2]}}. \tag{3c}$$

Because the dielectric interface at $y = -h$ is not a magnetic wall, we assume that the field produced by the structure shown in Fig. 2(a) can be obtained approximately by combining $\vec{E}_a$ and $\vec{E}_b$ linearly, that is

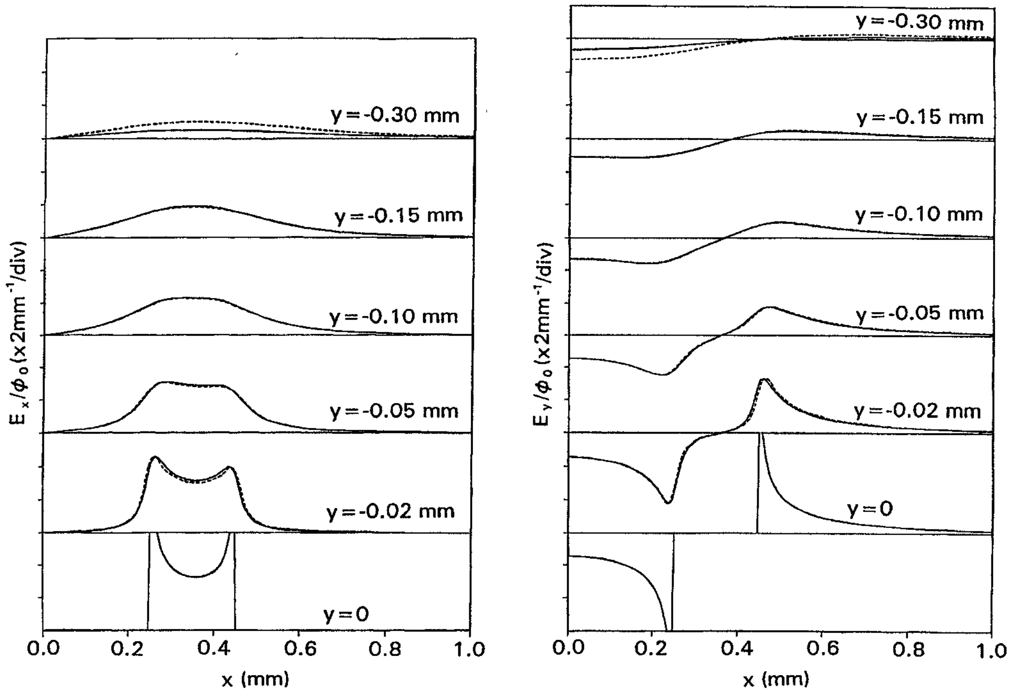
$$\vec{E} = \begin{cases} \vec{E}_b, & y > 0, \\ (\epsilon_{L_{eff}} - \epsilon_{r3})/\epsilon_{L_{eff}}\vec{E}_a + \epsilon_{r3}/\epsilon_{L_{eff}}\vec{E}_b, & 0 \geq y \geq -h, \\ \epsilon_{r3}/\epsilon_{L_{eff}}\vec{E}_b, & y < -h \end{cases} \tag{4a}$$

where $\epsilon_{L_{eff}}$ denotes the effective dielectric constant of the lower half-plane, and

$$\epsilon_{L\mathrm{eff}} = \epsilon_{r3} + (\epsilon_{r2} - \epsilon_{r3})\frac{K(k_1)K(k')}{K(k_1')K(k)}. \tag{4b}$$

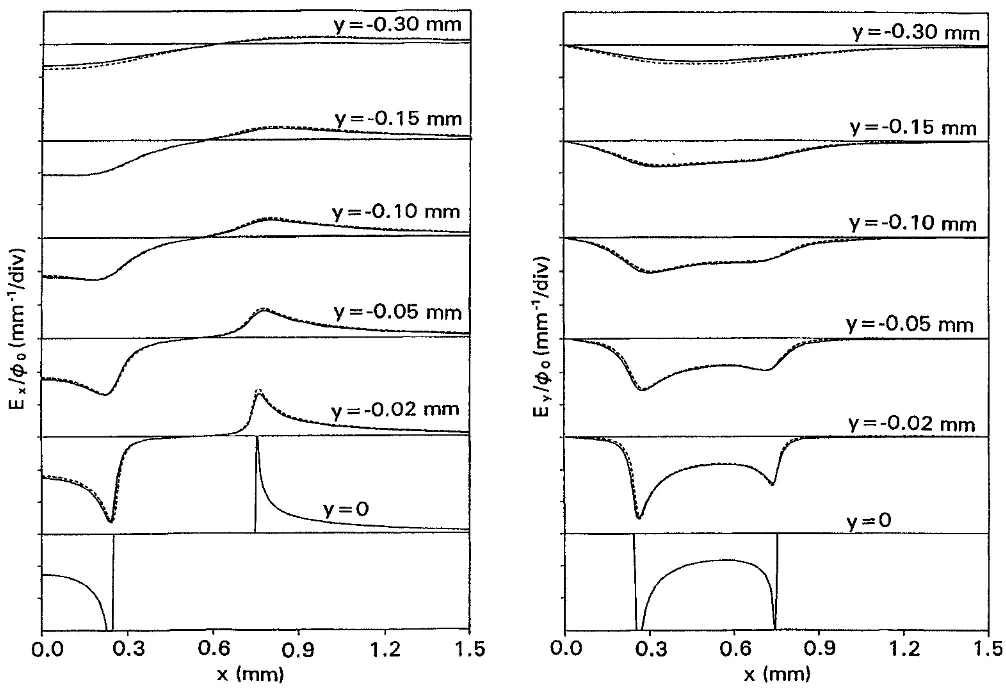
If the term $(\epsilon_{L_{eff}} - \epsilon_{r3})$ is replaced by $(\epsilon_{r2} - \epsilon_{r3})$, the continuity of the normal displacement is satisfied at $y = -h$, but the difference in tangent electric fields at the interface becomes larger, and this will lead to poorer accuracy. Since the interface at $y = -h$ is not a magnetic wall, $\epsilon_{L_{eff}}$ is used instead of ϵ_{r2} in our approximation. Although the solution does not strictly satisfy the boundary conditions at $y = 0$ and $y = -h$, the field variation tendencies toward the limitation of the structure parameters and dimensions are reasonable. Let $\vec{E}_2$ and $\vec{E}_3$ indicate the fields in domain II and III. From the properties of $\vec{E}_a$ and $\vec{E}_b$, it follows that this solution will satisfy the following requirements:

1) $E_x = 0$ and $E_y \neq 0$ on the electrodes.
2) $E_x \neq 0$ and $E_y = 0$ in the gaps.
3) $-\int_{x_G}^{x_c} \vec{E} \cdot dl = \phi_0$.
4) When $h \to 0$ or $\epsilon_{r2} \to \epsilon_{r3}, \vec{E}_2 \to \vec{E}_b$ and $\vec{E}_3 \to \vec{E}_b$.
5) When $h \to \infty, \vec{E}_2 \to \vec{E}_b$.
6) When $\epsilon_{r2} \to \infty, \vec{E}_2 \to \vec{E}_b$ and $\vec{E}_3 \to 0$.



(a) (b)

Fig. 3. Field distributions in (a) the x direction and (b) the y direction for coplanar waveguide with the following parameters: $a = 0.25$ mm, $b = 0.45$ mm, $h = 0.2$ mm, $\epsilon_{r1} = 1, \epsilon_{r2} = 20$, and $\epsilon_{r3} = 10$. Dashed line: the point match method; solid line: the present method.



(a) (b)

Fig. 4. Same as Fig. 3 but for the coplanar strips with the following parameters: $a = 0.25$ mm, $b = 0.75$ mm, $h = 0.2$ mm, $\epsilon_{r1} = 1, \epsilon_{r2} = 12.9$, and $\epsilon_{r3} = 10$.

The integration path starting from an arbitrary point x_G on the ground conductor and running to an arbitrary point x_C on the center electrode can be arbitrarily chosen in domain I and II. For the supported strips (CPS), the field can be obtained from (4a), but in which

$$\epsilon_{L\mathrm{eff}} = \epsilon_{r3} + (\epsilon_{r2} - \epsilon_{r3}) \frac{K(k_1')K(k)}{K(k_1)K(k')}. \tag{5a}$$

$$\vec{E}_a = \frac{\phi_0}{2K(k_1)}[-\vec{a}_x \mathrm{Re}(dW_1/dZ) + \vec{a}_y \mathrm{Im}(dW_1/dZ)], \quad (5b)$$

$$k_1 = \tanh(\pi a/2h)/\tanh(\pi b/2h), \quad (5c)$$

$$dW_1/dZ = \frac{(\pi/2h)}{\tanh(\pi a/2h)\cosh^2(\pi Z/2h)\sqrt{(1-Z_1^2)(1-k_1^2 Z_1^2)}}, \quad (5d)$$

$$Z_1 = \tanh(\pi Z/2h)/\tanh(\pi a/2h), \quad (5e)$$

$$\vec{E}_b = \frac{\phi_0}{2K(k))}[-\vec{a}_x \mathrm{Re}(dW/dZ) + \vec{a}_y \mathrm{Im}(dW/dZ)] \quad (5f)$$

k and dW/dZ are given in (3b) and (3c), and $k' = \sqrt{1-k^2}$.

III. Results

Our analytical expressions are applied to the supported coplanar waveguide considered in [5] and coplanar strips. The computed field profiles are plotted in Figs. 3 and 4. The results obtained using the point matching method [9] are also given in the figures for comparison. The series expansion including 400 terms is used and the calculated domain is 10 mm. From Fig. 3, one can see that the results for the case $y = -0.3$ mm are not good. However, this is unimportant because only the fields near the electrode gaps are of interest for the practical applications. In Figs. 3 and 4, the results at $y = 0$ obtained by the point matching method are not included because of the serious oscillations. The case $y = -h$ is not calculated since the normal electric field is not continuous. The comparison shows that the proposed analytical expressions give excellent descriptions for the field distributions, even for the case when the coplanar waveguide has a large dielectric difference between the substrate and the supporting material.

IV. Conclusion

In summary, we have derived analytical expressions for calculating the field distributions in two typical supported coplanar lines, including coplanar lines with finitely thick substrate. This method provides an accurate, simple and fast approach to calculating the field distributions in coplanar lines. Calculated results agree well with those obtained using the point matching method. Although only symmetric coplanar waveguide and coplanar strips have been discussed, our method can also be applied to the analysis of other coplanar transmission lines, such as covered supported coplanar waveguide, overlayed coplanar waveguide [5], and modified coplanar stripline [7]. Using the coordinate transformation [11], this method can be extended to analyze the coplanar lines fabricated on anisotropic substrate, which are widely used for integrated optical devices [12]. Hence the present methods are very useful for MMIC and other similar applications where field distributions are essential.

References

[1] C. Veyres and V. Fouad-Hanna, "Extension of the application of conformal mapping techniques to coplanar lines with finite dimensions," *Int. J. Electron.*, vol. 48, pp. 47–56, 1980.

[2] V. Fouad-Hanna and D. Thebault, "Theoretical and experimental investigation of asymmetric coplanar waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, pp. 1649–1651, 1984.

[3] G. Ghione and C. Naldi, "Coplanar waveguides for MMIC applications: effect of upper shielding, conductor backing, finite-extent ground planes, and line-to-line coupling," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 260–267, 1987.

[4] ——, "Analytical formulas for coplanar lines in hybrid and monolithic MIC's," *Electron. Lett.*, vol. 20, pp. 179–181, 1984.

[5] S. S. Bedair and I. Wolff, "Fast, accurate and simple approximate analytic formulas for calculating the parameters of supported coplanar waveguides for (M)MIC's," *IEEE Trans. Microwave Theory Tech.*, vol. 40, pp. 41–48, 1992.

[6] S. S. Gevorgian and I. G. Mironenko, "Asymmetric coplanar-strip transmission lines for MMIC and integrated optic applications," *Electron. Lett.*, vol. 26, pp. 1916–1918, 1990.

[7] J. S. Mclean and T. Itoh, "analysis of a new configuration of coplanar stripline," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-40, pp. 772–774, 1992.

[8] N. H. Zhu, Z. Q. Wang, and W. Lin, "On the accuracy of analytical expressions for calculating the parameters of coplanar strips on a finitely thick substrate," *Microwave Opt. Technol. Lett.*, vol. 7, pp. 160–164, 1995.

[9] D. Marcuse, "Electrostatic field of coplanar lines computed with the point matching method," *IEEE J Quantum Electron.*, vol. QE-25, pp. 939–947, 1989.

[10] T. N. Chang and Y. C. Sze, "Flexibility in the choice of Green's function for the boundary element method," *IEEE Trans. Microwave Theory Tech.*, vol. 42, pp. 1973–1977, 1994.

[11] O. G. Ramer, "Integrated optic electrooptic modulator electrode analysis," *IEEE J. Quantum Electron.*, vol. QE-18, pp. 386–392, 1982.

[12] R. C. Alferness, "Waveguide electrooptic modulators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 1121–1137, 1982.

Network Representation and Transverse Resonance for Layered Chirowaveguides

Xu Shanjia and Du Kai

Abstract— **This paper presents an equivalent network method for dispersion analysis of general chirowaveguides. First, wave propagation in each homogeneous layer is represented by two pairs of transmission lines, the matrix wave impedance is defined. Next, the transformation properties of the input impedance are established. It is then demonstrated that the transverse resonance condition involving the previously obtained matrix impedance leads to the dispersion equation for the waveguides. The numerical results show that this network approach is feasible and practicable.**

I. Introduction

In recent years, a number of papers have appeared in the literature [1]–[5] attacking different kinds of chirowaveguides and the guided wave properties in these new waveguides. Creative use of chiral material, especially for integrated circuits, has been envisioned. The multilayered planar chirowaveguides will become the building block and thus is the foundation for analysis of a large class of chiral guided wave structures. In our opinion, the existing methods are not general enough to enable the interested worker in the microwave and millimeter-wave areas to readily solve for the propagation constant

Manuscript received June 10, 1995; revised April 19, 1996.

The authors are with the Department of Electronic Engineering and Information Science, University of Science and Technology of China, Hefei, Anhui, 230027, P. R. China.

Publisher Item Identifier S 0018-9480(96)05638-4.